
**Limit Order Book:**
A limit order book is an electronic record of unexecuted limit orders on various price levels

**Nasdaq Itch Data:**
Event feed data contains all messages of events occurring in Nasdaq Stock Exchange, from which we can construct an evolving limit order book

**A Trading Problem:**
Goal: Buy a share of stock over a time window \([0, T]\)
Implementation Shortfall: \(p_E - p_M(0)\)

- **Market Order:**
  \[ \mathbb{E}[IS_1] = p_A(0) - p_M(0) = \text{half–spread} = S \]

- **Limit Order:**
  \[ \mathbb{E}[IS_2] = p(W \leq T)p_B(0) - p_M(0) + p(W > T)\mathbb{E}[p_A(T) - p_M(0)|W > T] \]

**Threshold Policy:**
\[ p(W \leq T) = \frac{\mathbb{E}[p_A(T) - p_M(0)|W > T] - S}{\mathbb{E}[p_A(T) - p_M(0)|W > T] + S} \]

**Tradeoff:**
Market Order – Certainty of Execution, Price Disadvantage
Limit Order – Uncertainty of Execution, Price Premium
Fill Probability – Essential of evaluating this tradeoff and need to be estimated

2. Limit Order Fill Time Simulation
We simulate synthetic limit orders to top price levels into limit order books and track their positions. When it gets to the top of the book, we record its fill time.

3. Recurrent Neural Network – Density Estimation
We divide positive half-line into pre-determined intervals and estimate a constant hazard rate on each interval.

![Output Layer Diagram]

The set of hazard rates \(\lambda_i\)\(i=1,...,N\) uniquely determines a distribution with close form density. From these density, fill probability can be computed for any time-threshold.

4. Results and Contribution
**Fill Probability:**

<table>
<thead>
<tr>
<th>Predictive Performance (AUC)</th>
<th>(t = 1) Min</th>
<th>(t = 5) Min</th>
<th>(t = 10) Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logistic Regression</td>
<td>0.62</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Bucket Estimator</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>RNN</td>
<td>0.71</td>
<td>0.67</td>
<td>0.66</td>
</tr>
</tbody>
</table>

**Implementation Shortfall:**

<table>
<thead>
<tr>
<th>Average IS (ticks)</th>
<th>(t = 1) Min</th>
<th>(t = 5) Min</th>
<th>(t = 10) Min</th>
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</thead>
<tbody>
<tr>
<td>Logistic Regression</td>
<td>0.321</td>
<td>0.198</td>
<td>0.163</td>
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<tr>
<td>Bucket Estimator</td>
<td>0.316</td>
<td>0.192</td>
<td>0.159</td>
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<tr>
<td>RNN</td>
<td>0.281</td>
<td>0.176</td>
<td>0.148</td>
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</tbody>
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5. Selected References