How to Play Strategically in Fantasy Sports (and Win)

Martin B. Haugh¹
Raghav Singal²

¹Imperial College Business School, Imperial College, ²IEOR, Columbia University

Introduction

- Daily Fantasy Sports (DFS) is a multi-billion dollar industry
- Millions of annual users at two major websites (FanDuel and DraftKings)
- Regular appearance of articles discussing DFS in mainstream media

Benchmark: mean, sd, p(loss) = 86.30, 694, 0.46

Goal: Provide a coherent framework for constructing portfolios

- Previous work by Hunter, Vielma, and Zaman [2]
  - They approximate the payoff structure by winner-takes-all
    - Non-convex objective
    - Do not consider opponents’ behavior
    - Approximately maximize the probability of winning

- Need Monte Carlo samples of two underlying uncertainties

Problem Formulation

- P athletes with performance δ ∈ R^P (mean μ, variance-covariance Σ)
- Decision: Choose a portfolio w ∈ {0, 1}^P of athletes
- Constraints: W is the set of feasible portfolios (budget, diversity, etc.)
  - Opponent portfolios: O athletes with portfolio W_o := \{w_o\}_o=1^O
  - Opponent points total: F := \sum w_o \delta
  - Portfolios ranked according to their points and rewarded accordingly

- Reward structure: Double-up and top-heavy
  - Double-up: Top r portfolios each earn a payoff of R dollars
  - Top-heavy: Top few ranks win R1, next few win R2 < R1 and so on

- We can submit up to N portfolios but we start with N = 1 (for simplicity)
- Maximizing expected reward can be formulated as follows:

\[
\max_{w} \mathbb{E} \left[ w^T \delta \right]
\]

- \(G^{(r)}(W_o, \delta)\) is the rth order statistic of \(\{G_o\}_o=1^O\) and we define \(r' := O + 1 - r\)
- Same as maximizing the probability we exceed a stochastic benchmark
- Non-trivial optimization problem
  - Discrete decision: Either pick an athlete or not
  - Non-convex objective: Maximizing probability we win
  - Next: How do we model opponents’ \(w_o\)?

Mean-Variance Optimization and Stochastic Benchmarks

- Our optimization model can be stated as follows:

\[
\max_{w} \mathbb{E} \left[ w^T \delta \right] - \frac{1}{2} w^T \Sigma w
\]

- Assume \(Y_w := w^T \delta - G^{(r)}\) is normally distributed
- Can find the optimal portfolio \(w^*\) using a result from Morton et al. [3]
- Intuition: Control a normal random variable using its mean and variance
  - If mean is below the benchmark, maximize combination of mean + variance
  - Else: maximize combination of mean − variance
  - Need to solve a series of Binary Quadratic Programs to find \(w^*\)

- For \(N > 1\), portfolio replication is optimal (for double-up)
- Similar approach works for top-heavy under some additional assumptions
  - Various parameters to be estimated, for example, Cov(δ_o, G^{(r)})

Monte Carlo and Order Statistics

- We use Monte Carlo to estimate input parameters for optimization
- Need Monte Carlo samples of two underlying uncertainties, \(\delta\) and \(W_o\)
- Sampling \(W_o\) is computationally expensive (since \(O\) is big)

- Observations
  - \(W_o\) only affects the stochastic benchmark \(G^{(r)}\)
  - \(G_o\) independent and identically distributed for \(o = 1, \ldots, O\)

- Idea: Use theory of order statistics [1] to sample \(G^{(r)}\) efficiently

\[
G^{(r)}(1-p_o) = F_{G^{(r)}}^{-1}(p_o) = \frac{r_o}{O}
\]

- \(F_{G^{(r)}}^{-1}(1-p_o)\) denotes the inverse CDF function of \(G^{(r)}\)

- Approximately compute \(G^{(r)}\) and \(F_{G^{(r)}}^{-1}(1-p_o)\)
- \(p_o\) varies from 0.001 to 0.999
- \(r_o\) varies from 1 to 15

- Sample \(G^{(r)}\) with \(p_o\)

- Can find the optimal portfolio \(w^*\)

- Value of insider trading large when one is not strategic

Results

- Participated in contests on FanDuel during the 2017-18 NFL season
- Benchmark model similar to Hunter, Vielma, and Zaman [2]
- Invested $50 per week for each of the two models in a top-heavy contest
- ROI of over 350% in 17 weeks for the strategic model

Conclusion and Future Research

- Developed a coherent framework for constructing fantasy sports portfolios
- Explicitly modeled opponents and leveraged mean-variance theory from finance
- Demonstrated the value of our approach in real-world contests
- Provided insights into the value of insider trading
- Future research: Other sports, real-time parameters, strategic opponents

References