We study Bayesian algorithms in this frequentist setting. An algorithm begins with a multivariate normal prior $N(\mu, \Sigma)$. The objective is to optimize $\theta = (\theta_1, \ldots, \theta_k)$.

**Expected Improvement (EI) Measure.**

$$V_{n,i} = \mathbb{E}_{\phi \sim N(\mu, \Sigma)} \left[ (\hat{\theta}_i - \mu_{n, i})^2 \right]$$

where $I_n^* = \max_{i \in [k]} I_n$.

- \text{Can be computed in closed form.}
- \text{Increase in posterior mean and posterior variance.}

**EI sampling rule measures the arm**

$$I_n = \arg \max_{i \in [k]} V_{n,i}.$$

**EI is too greedy:** allocate only $O(\log n)$ to suboptimal arms (Ryzhov (2016)).

**Comparative Expected Improvement (CEI) Measure.**

$$V_{n,i,j} = \mathbb{E}_{\phi \sim N(\mu, \Sigma)} \left[ (\hat{\theta}_i - \hat{\theta}_j)^2 \right].$$

- \text{Integrate over the uncertain quality of both arms.}
- \text{Can be computed in closed form as well.}

**Top-Two Expected Improvement (TTEI) Algorithm.**

- **TTEI sampling rule measures the arm**

$$I_n = \left\{ \begin{array}{ll} I_n^{(1)} &= \arg \max_{i \in [k]} V_{n,i} & \text{w.p. } \beta_n, \\ I_n^{(2)} &= \arg \max_{i \in [k]} V_{n,i,L} & \text{w.p. } 1 - \beta_n. \end{array} \right.$$  

**TTEI stopping rule stops when**

$$\sum_{i \in [k]} V_{n,i,L} \leq \eta_n \beta_n$$

where LHS bounds the expected shortfall of $I_n^*$ from above.

**Problem Complexity Measure**

Problem complexity term $C^*(\theta)$ measures the hardness of identifying the best arm under a problem with arm means $\theta = (\theta_1, \ldots, \theta_k)$.

$$C^*(\theta) = \frac{\max_{i \in [k]} \min_{w \neq w(i)} \left( \frac{(\theta_i - \theta_j)^2}{2\sigma^2(1/\sqrt{w} + 1/w)} \right)}{\sqrt{\log(1/\delta)}}$$

In practice, $\{\beta_n\}$ can be set to a fixed $\beta \in (0, 1)$.

$$C^*_n(\theta) = \frac{\max_{i \in [k]} \min_{w \in [2^N]} \left( \frac{(\theta_i - \theta_j)^2}{2\sigma^2(1/\sqrt{w} + 1/w)} \right)}{\sqrt{\log(1/\delta)}}$$

For any $\beta \in (0, 1)$ and $\theta$,

$$C^*_n(\theta) \leq \min_{\beta \in (0, 1)} \left\{ \beta^2 \frac{C^*(\theta)}{\beta^2} \right\} = \frac{C^*(\theta)}{\beta^2}.$$  

In particular, $C^*_n(\theta) \leq 2C^*(\theta)$. This demonstrates a surprising degree of robustness to $\beta$.