Unsupervised Sparse-view Backprojection via Convolutional and Spatial Transformer Networks

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Introduction
We use convolutional neural networks (CNNs) and a Spatial Transformer Network to solve the backprojection problem. We evaluated our algorithm using computed tomography (CT) images of the human chest and show that our algorithm significantly out-performs filtered backprojection when the projection angles are very sparse, as well as when the sensor characteristics vary for different angles.

Problem Modeling

![Clean image vs single-view back projections for 8 angles](Image)

![Sinogram](Image)

**The relationship between ground truth image $S$ and its projections is Radon transformation:**

$$\text{Radon}(S, \theta) = R_{\theta}(I(\theta), \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(x, y) \delta(t(\theta) - x \cos \theta - y \sin \theta) \, dx \, dy$$

A sinogram, a standard data structure to store projections, is defined as the mnx matrix made of n projections from m different angles. Instead of using sinograms directly as input to a CNN we construct single-view backprojections.

Method

**Generator-Projector framework for back-projection:** The generator predicts the reconstruction from the sparsely measured sinograms, while the projector maps the reconstruction back to the original sinograms.

- Sinograms are converted to single-view backprojections and fed to a multi-layer CNN backprojection generator.
- The Spatial Transform Linear Projector applies the Radon transform of correspondent angles to backprojection reconstruction to regenerate the sinogram prediction $\tilde{p}_i = \text{Radon}(\tilde{S}, \theta_i)$
  - The grid generator transforms the original regular spatial grid of the reconstruction to a sampling grid.
  - The sampler produces the sampled transformed data from the reconstruction at the grid points.
  - Then trainable linear mapping, $p'_i = w_i \tilde{p}_i + b_i$, compensates for possible sensor non-uniformity.
  - We include an l1-norm of predicted backprojection reconstruction $\tilde{S}$ to impose a sparse reconstruction.

$$\text{Objective function is } f(k, w, b) = \frac{1}{n} \sum_{k=1}^{n} \|p_k - p'_k\|_2 + \alpha \|\tilde{S}\|_1.$$  

Results

We evaluate our algorithm using 43 human chest CT scans. The Radon transformation was applied to each slice of the CT data with and without sensor non-uniformity respectively.

Fig.3a compares our algorithm to filtered backprojection assuming sensor uniformity (i.e. fixed $w_i = 1$ and $b_i = 0$ for STN). The improved performance is especially apparent in very sparse cases - 2/4-angle projection.

A comparison of reconstruction PSNR between our algorithm and filtered backprojection is shown in Fig.3c. Our algorithm shows significantly better PSNR than filtered backprojection.

Fig.3b shows reconstruction results for our algorithm compared with filtered backprojection for the case of sensor non-uniformity. Filtered backprojection cannot adjust to sensor non-uniformity and shows strong artifacts while our algorithm can provides reasonable reconstructions.

Fig.3d shows the correlation coefficients of our algorithm and filtered backprojection. Our algorithm shows significantly better performance.

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Reference
