Distributionally Robust Mean-Variance Portfolio Selection with Wasserstein Distances

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1. Introduction

We revisit the Markovitz's mean-variance portfolio selection model by considering a distributionally robust version (Figure 1.1), where the region of distributional uncertainty is centered at the empirical measure and the distance between probability measures is defined by Wasserstein distance (Figure 1.2).

$$\min_{\phi \in \mathcal{F}_{\delta,\bar{\alpha}}(n)} \max_{\mathbb{P} \in \mathcal{U}_{\delta}(\mathbb{P}_n)} \{\phi^{\mathrm{T}} \operatorname{Var}_{\mathbb{P}}(R)\, \phi\}$$

$$\mathcal{U}_{\delta}(\mathbb{P}_n) := \{\mathbb{P} : D_c(\mathbb{P}, \mathbb{P}_n) \leq \delta\}$$

$$\mathcal{F}_{\delta,\bar{\alpha}}(n) = \left\{\phi : \phi^{\mathrm{T}} 1 = 1, \min_{\mathbb{P} \in \mathcal{U}_{\delta}(\mathbb{P}_n)} \left[\mathbb{E}_{\mathbb{P}}(\phi^{\mathrm{T}} R)\right] \geq \bar{\alpha}\right\}$$

Figure 1.1. Distributionally Robust Mean-Variance Model

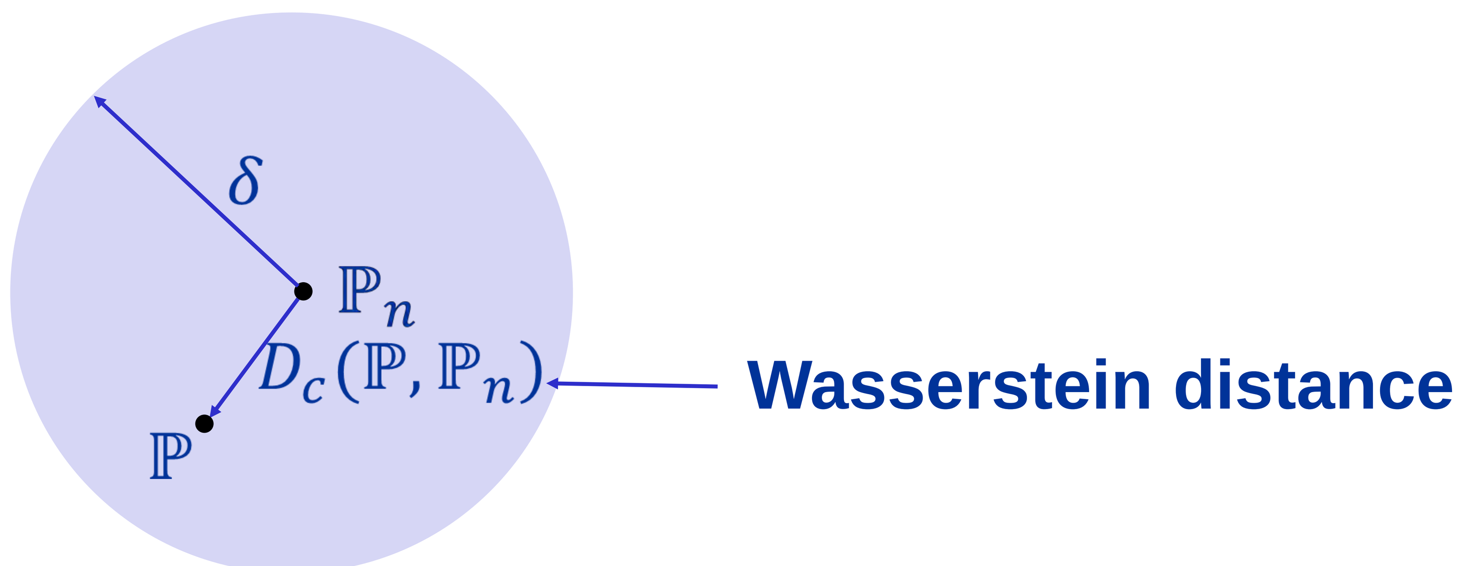

Figure 1.2. Graph of uncertainty set

2. An Equivalent Data-Driven Model

We reduce this problem into an empirical variance minimization problem with an additional regularization term (Figure 2.1). This problem is a trackable convex optimization problem and can be further simplified to be a Second Order Cone Programming (SOCP) problem. Moreover, we extend the recently developed inference methodology (Blanchet, Kang and Murthy. 2016) to our setting in order to select the size of distributionally uncertainty as well as the associated robust target return rate in a data-driven way:

$$\min_{\phi \in \mathcal{F}_{\delta,\bar{\alpha}}(n)} \left(\sqrt{\phi^{\mathrm{T}}\, Var_{\mathbb{P}_n}(R)\, \phi} + \sqrt{\delta} \|\phi\|_p\right)^2$$

$$\mathcal{F}_{\delta,\bar{\alpha}}(n) = \left\{\phi \in \mathbb{R}^d : \phi^{\mathrm{T}} 1 = 1, \mathbb{E}_{\mathbb{P}_n}(\phi^{\mathrm{T}} R) \geq \bar{\alpha} + \sqrt{\delta} \|\phi\|_p\right\}$$

Figure 2.1. The Equivalent Data-Driven Model

3. Backtesting Results

We have done extensive backtesting results on S&P 500 that compare the performance of our model with those well-know models including Fama-French model and the Black-Litterman model.

We backtested for the time period January 2000—December 2016. At the beginning of 2000, we randomly chose 20 stocks from S&P 500 and update the portfolio allocation each month. Then we repeat the experiment for 100 times to obtain the average portfolio wealth plot (Figure 3.1) and the histograms of annualized returns (Figure 3.2).

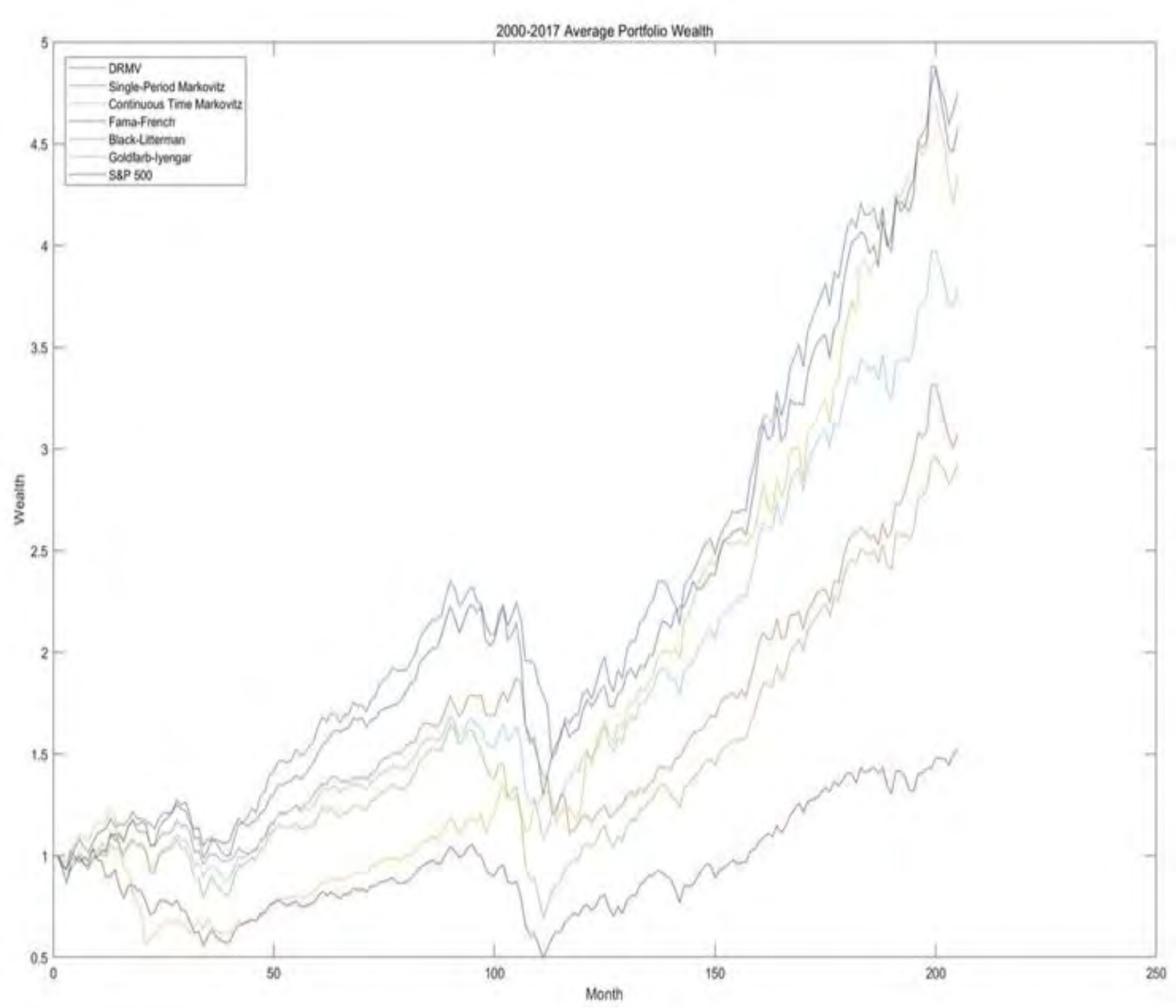

Figure 3.1. Average portfolio wealth over 100 experiments

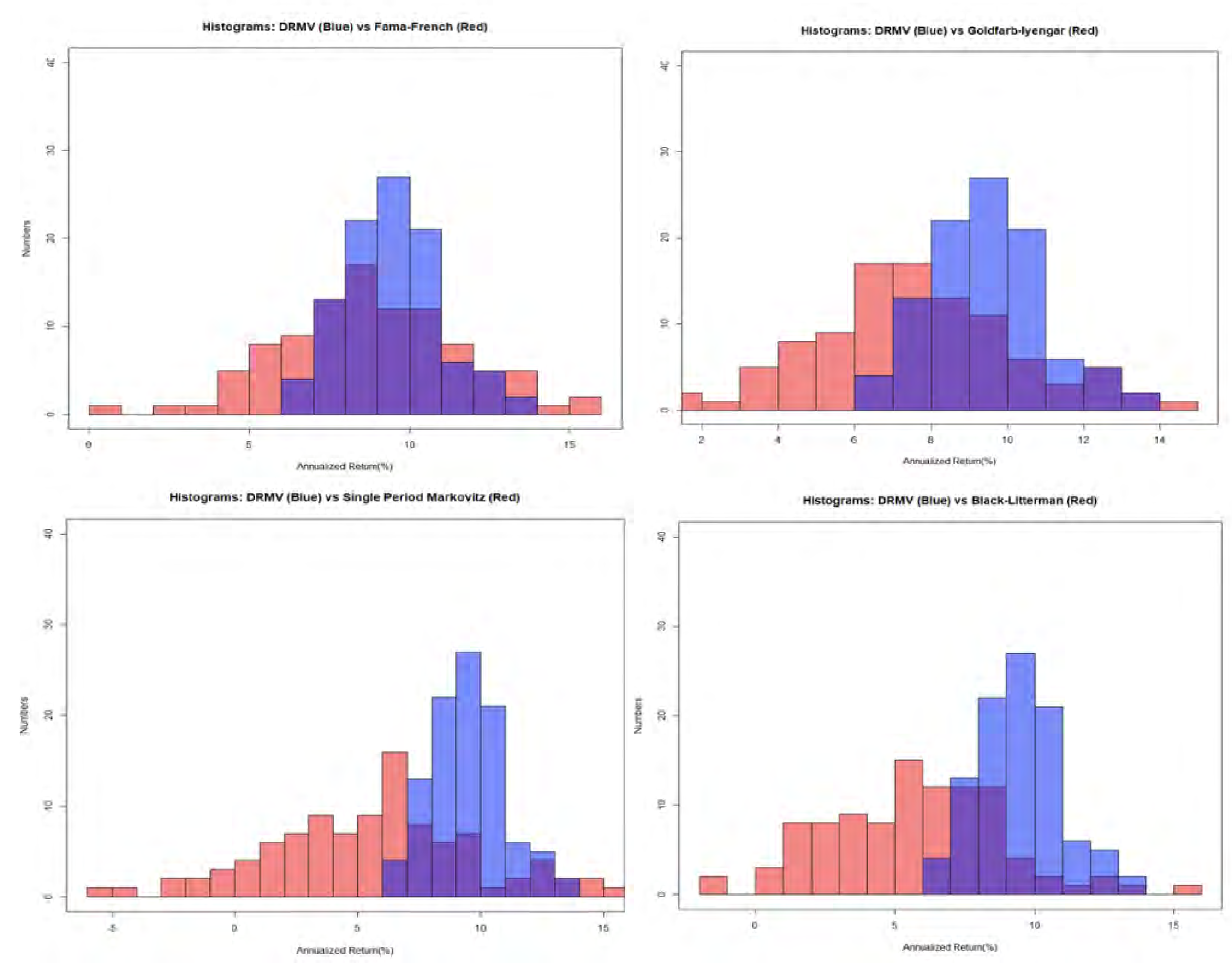

Figure 3.2 Histograms of Annualized return over 100 experiments

4. Conclusion

We have provided a data-driven distributionally robust theory for Markovitz's mean-variance portfolio selection. The robust model can be solved via a non-robust one based on the empirical probability measure with an additional regularization term. The size of the distributonal uncertainty region is not exogenously given; rather it is informed by the return data in a scheme which we have developed in this project.

Acknowledgments

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References

Blanchet, J., Kang, Y., and Murthy, K. (2016) Robust Wasserstein Profile Inference and Applications to Machine Learning. arXiv: https://arxiv.org/abs/1610.05627