A Queueing Model for the Inpatient Labor and Deliver Unit

Yue Hu*, Jing Dong*, Ohad Perry*
+ Columbia Business School
* Northwestern University

Introduction

We take a queueing approach to analyze and model the patient flow dynamics in the labor and deliver (LD) units. We acquire and explore the patient flow data from a large urban academic hospital with 110 beds in its LD unit, where 8246 (73.6%) vaginal-delivery patients and 2965 (26.4%) C-section patients were treated in the year 2016-2017.

Figure 1. Typical patient flow dynamics in the LD units

The Data and Model

Among the complex patient-flow dynamics, we identify and incorporate the following salient characteristics of the related queueing processes on the postpartum floor in the LD unit.

Data: Hourly bed request rate

Data: Day-of-week bed request rate

Model: Two-class infinite server model with time varying arrival rate (Chan et al 2016)

Data: Patient length of stay (LOS) in days

Data: Discharge delay across the day

Model: Two time scale LOS distribution (Shi and Dai 2016, Dong and Perry 2018)

Medical requirement (number of nights): two point distribution with 1 or 2 nights for vaginal delivery, 3 or 4 nights for C-section

Discharge delay (hours)

An example patient sojourn in the system is shown below in Figure 2:

Figure 2. A Queueing model for the postpartum floor in the LD unit

Discussion

Model validation: Using the queueing model, we derive that the occupancy level at any time t follows a Poisson distribution with mean m(t), which is periodic with period equal to a week. In Figure 3, we compare the model prediction with the data. The predicted average occupancy level matches the mean of the data well.

Two-class M^{(1)} / G / ∞ queue:

\[ X(t) = \text{Poisson}(m(t)) \]

where \( m(t) = p_1(1) \int_0^t \lambda_1(s)P(D > t - s)ds + \int_0^t \lambda_2(s)ds \)

\( + p_2(2) \int_0^t \lambda_3(s)P(D > t - s)ds + \int_0^t \lambda_4(s)ds \)

Day-of-week effect: We explore the day-of-week effect by comparing model outputs with data for models with periodic arrival rate over a day and over a week. Incorporating the day-of-week effect does not change the mean-level prediction, but boosts the predicted variance of the occupancy level to make it closer to the observed value in the data.

Period = 1 day

\[ \bar{X}(t) = \sum \lambda_i(t) \]

\[ X(t) \sim \text{Poisson}(\bar{m}(t)) \]

where \( \bar{m}(t) = \int_0^t \bar{X}(t)F(t - s)ds \)

Period = 1 week

Let \( D \) denote the day of the week, then

\( P(D = t) = \frac{1}{7} \)

Denote \( E[X(t;D) | D = t] = m(t) \). Then

\[ E[X(t;D)] = E[E[X(t;D) | D]] = \sum_{t=1}^7 m(t) = \bar{m}(t) \]

Mean matches data well.

\[ Var(X(t;D)) = Var(E[X(t;D) | D]) + Var(E[X(t;D) | D]) \]

\[ = \bar{m}(t) + \frac{1}{7} \sum_{t=1}^7 (m(t) - \bar{m}(t))^2 > \bar{m}(t) \]

Both the mean and variance matches data well.

References

Chan, C. W., Dong, J., & Green, L. V. (2016). Queues with time-varying arrivals and inspections with applications to hospital discharge policies. Operations Research, 65(2), 469-495.
