A Queueing Model for the Inpatient Labor and Deliver Unit

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Introduction

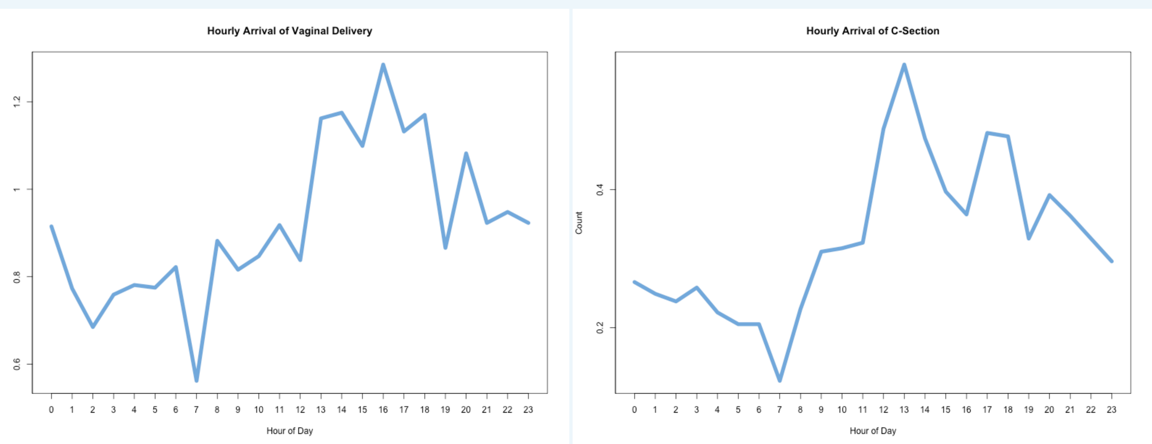
We take a queueing approach to analyze and model the patient flow dynamics in the labor and deliver (LD) units. We acquire and explore the patient flow data from a large urban academic hospital with 110 beds in its LD unit, where 8246 (73.6%) vaginal-delivery patients and 2965 (26.4%) C-section patients were treated in the year 2016-2017.

Figure 1. Typical patient flow dynamics in the LD units

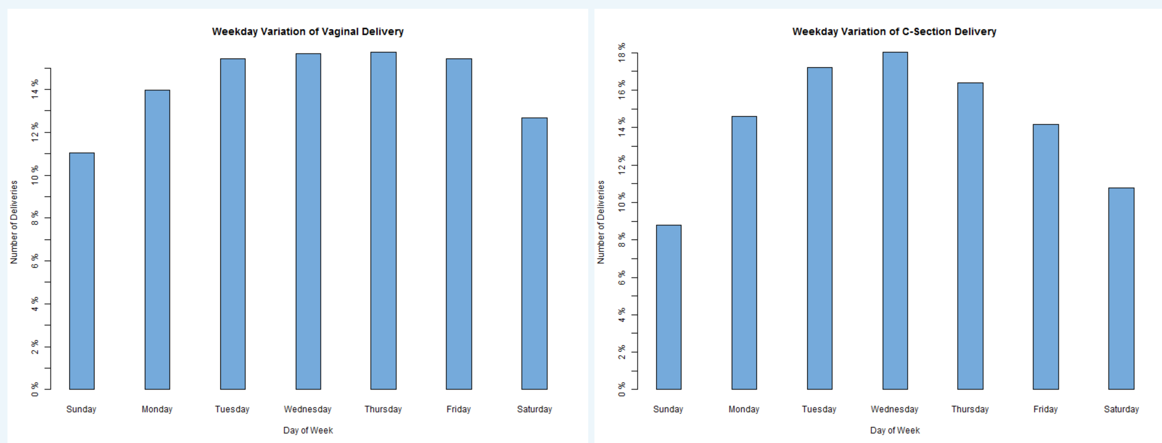
The Data and Model

Among the complex patient-flow dynamics, we identify and incorporate the following four salient characteristics of the related queueing processes on the *postpartum* floor in the LD unit.

❖ **Data:** Hourly bed request rate

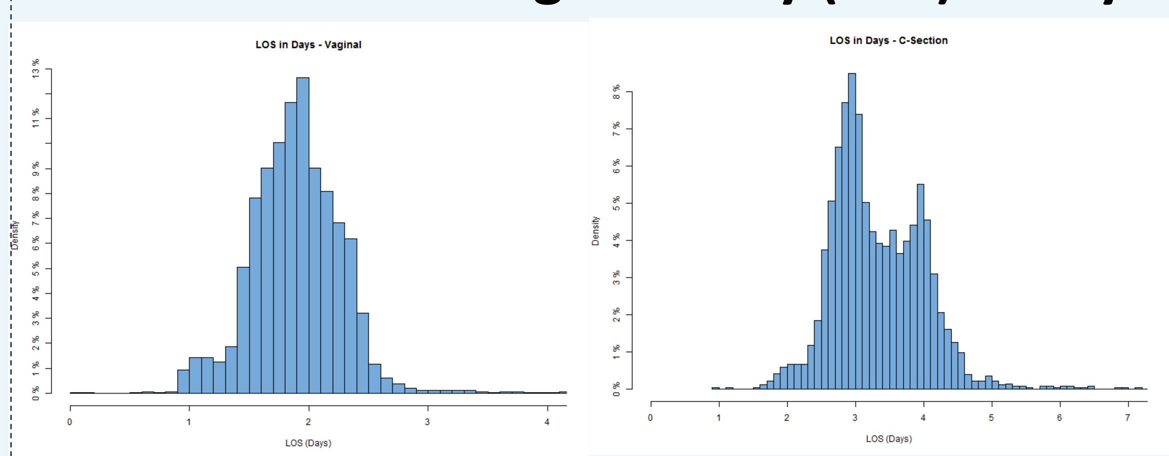


❖ **Data:** Day-of-week bed request rate

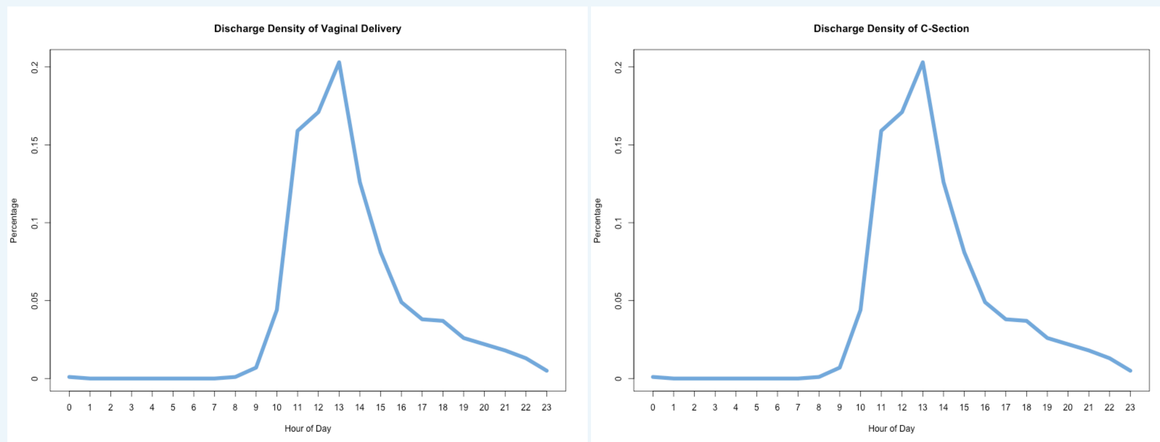


➢ **Model:** Two-class infinite server model with time varying arrival rate (Chan et al 2016)

❖ **Data:** Patient length of stay (LOS) in days



❖ **Data:** Discharge delay across the day



➢ **Model:** Two time scale LOS distribution (Shi and Dai 2016, Dong and Perry 2018)

<u>Medical requirement (number of nights)</u>: two point distribution with 1 or 2 nights for vaginal delivery, 3 or 4 nights for C-section **+** <u>Discharge delay (hours)</u>

An example patient sojourn in the system is shown below in Figure 2:

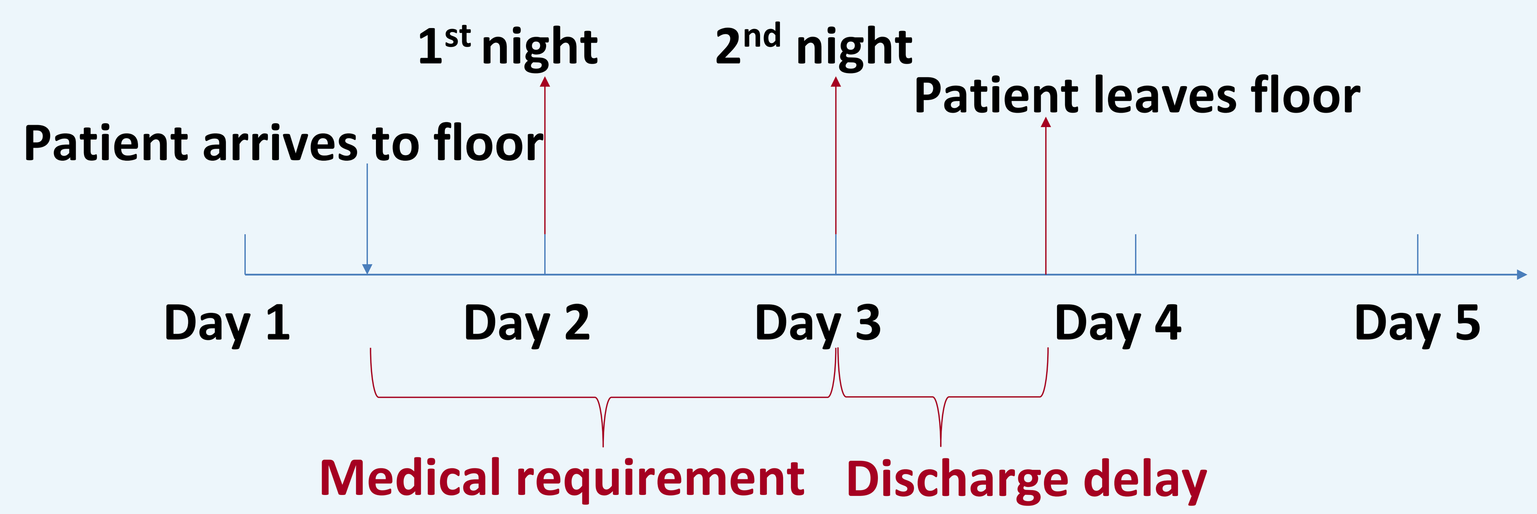


Figure 2. A Queueing model for the postpartum floor in the LD unit

Discussion

➢ <u>Model validation:</u> Using the queueing model, we derive that the occupancy level at any time t follows a Poisson distribution with mean m(t), which is periodic with period equal to a week. In Figure 3, we compare the model prediction with the data. The predicted average occupancy level matches the mean of the data well.

Two-class $M_p(t)/G/\infty$ queue:

$X_v(t) \sim \text{Poisson}(m_v(t))$

$$\text{where } m_v(t) = p_v(1)\left(\int_{t-2}^{\lfloor t\rfloor}\lambda_v(s)P(D>t-\lceil s\rceil)ds + \int_{\lfloor t\rfloor}^{t}\lambda_v(s)ds\right) + p_v(2)\left(\int_{t-3}^{\lfloor t\rfloor-1}\lambda_v(s)P(D>t-\lceil s\rceil-1)ds + \int_{\lfloor t\rfloor-1}^{t}\lambda_v(s)ds\right)$$

$X_c(t) \sim \text{Poisson}(m_c(t))$

$$\text{where } m_c(t) = p_c(3)\left(\int_{t-4}^{\lfloor t\rfloor-2}\lambda_c(s)P(D>t-\lceil s\rceil-2)ds + \int_{\lfloor t\rfloor-2}^{t}\lambda_c(s)ds\right) + p_c(4)\left(\int_{t-5}^{\lfloor t\rfloor-3}\lambda_c(s)P(D>t-\lceil s\rceil-3)ds + \int_{\lfloor t\rfloor-3}^{t}\lambda_c(s)ds\right)$$

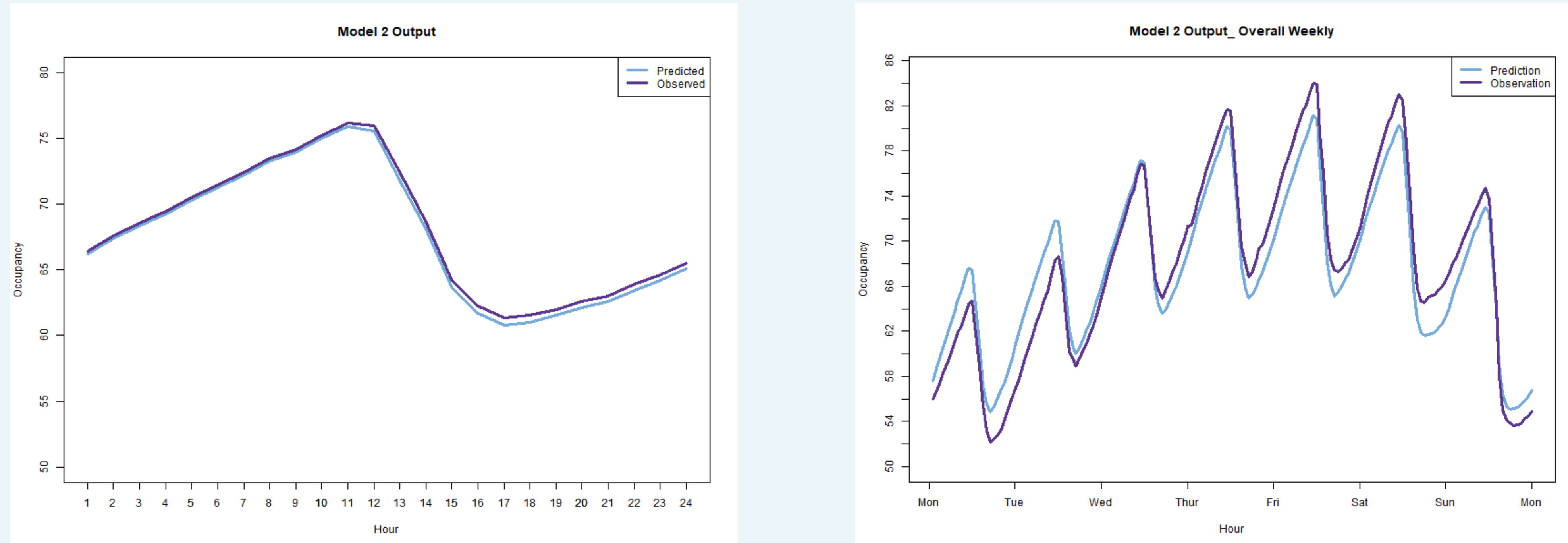


Figure 3. Comparison of the predicted (blue) and observed (purple) occupancy level over hourly (left) and daily (right) time scale

➢ <u>Day-of-week effect:</u> We explore the day-of-week effect by comparing model outputs with data for models with periodic arrival rate over a day and over a week. Incorporating the day-of-week effect does not change the mean-level prediction, but boosts the predicted variance of the occupancy level to make it closer to the observed value in the data.

❖ **Period = 1 day**

$$\bar{\lambda}(t) = \sum_{i=1}^{7}\lambda_i(t)$$

$$X(t) \sim \text{Poisson}(\bar{m}(t)) \text{ where } \bar{m}(t) = \int_{-\infty}^{t}\bar{\lambda}(t)F(t-s)ds$$

Mean matches data well.
Variance is lower than that in data.

❖ **Period = 1 week**

Let D denote the day of the week, then $P(D = i) = 1/7$ for i = 1, 2, …, 7.
Denote $E[X(t;D)\,|\,D=i] = m_i(t)$. Then

$$E[X(t;D)] = E[E[X(t;D)\,|\,D]] = \frac{1}{7}\sum_{i=1}^{7} m_i(t) = \bar{m}(t)$$

$$Var(X(t;D)) = E[Var(X(t;D)\,|\,D)] + Var(E[X(t;D)\,|\,D]) = \bar{m}(t) + \frac{1}{7}\sum_{i=1}^{7}(m_i(t)-\bar{m}(t))^2 > \bar{m}(t)$$

Both the mean and variance matches data well.

References

Chan, C. W., Dong, J., & Green, L. V. (2016). Queues with time-varying arrivals and inspections with applications to hospital discharge policies. *Operations Research*, *65*(2), 469-495.

Dai, J. G., & Shi, P. (2017). A two-time-scale approach to time-varying queues in hospital inpatient flow management. *Operations Research*, *65*(2), 514-536.

Dong, J., & Perry, O. (2018). Queueing Models for Patient-flow Dynamics in Inpatient Wards.