Generative Adversarial Networks (GANs)

- Training data is a set of data items $X = \{x_i\}_{i=1}^n$, where each $x_i$ is drawn from an unknown data distribution $\mathcal{X}$.
- A generative network $G$ learns to map from a known latent space distribution $\mathcal{Z}$ (typically a standard Gaussian distribution) to $\mathcal{X}$.
- Training is accomplished by introducing an additional network $D$ whose goal is to distinguish from the generated samples to the real data items drawn from $\mathcal{X}$.
- The generator $G$ is trained by solving a minimax optimization with the following objective.

$$L_{\text{gan}}(G, D) = \mathbb{E}_{x \sim \mathcal{X}}[\log D(x)] + \mathbb{E}_{z \sim \mathcal{Z}}[\log(1 - D(G(z)))].$$

This objective is minimized over $G$ and maximized over $D$.

The objective is optimized when the distribution of $G(z)$ is the same as LPDD of $\mathcal{X}$.

Mode Collapse

- Mode collapse is one of the most prominent issues in optimizing GANs.
- It is a phenomenon that a GAN can only capture a few modes of $\mathcal{X}$.
- The generated samples lack the diversity as shown in the real dataset.

Modes in Metric Space

We consider the geometric interpretation of modes:

- The modes of a data distribution should be viewed under a specific distance metric of data items.
- Different metrics may lead to different partitions of modes.
- We address the problem of mode collapse in a general metric space.

Logarithmic Pairwise Distance Distribution (LPDD). We propose to use the pairwise distance distribution of data items to reflect the mode structure in a dataset.

- Consider a metric space $(\mathcal{M}, d)$, and a distribution $\mathcal{X}$ over $\mathcal{M}$.
- Two independent samples $x, y$ are drawn from $\mathcal{X}$, and $\eta = \log(d(x, y))$.
- We call the distribution of $\eta$ conditioned on $x \neq y$ the logarithmic pairwise distance distribution (LPDD) of $\mathcal{X}$.

Then, ideally, we want to solve:

$$\min_{G} \max_{D} \mathbb{E}_{x \sim \mathcal{X}}[\log D(x)] + \mathbb{E}_{z \sim \mathcal{Z}}[\log(1 - D(G(z)))].$$

s.t. LPDD of $G(z)$ is as the same as LPDD of $\mathcal{X}$.

Latent Space Distribution

We question the commonly used multivariate Gaussian that generates random vectors for the generator network. In the presence of separated modes, drawing random vectors from a single Gaussian may lead to arbitrarily large gradients of the generator, and a better choice is by using a mixture of Gaussians.

**Theorem.** (Bourgain’s theorem) Consider a finite metric space $(Y, d)$ with $m = |Y|$. There exists a mapping $g : Y \rightarrow \mathbb{R}^k$ for some $k = O(\log^2 m)$ such that $\forall y, y' \in Y, d(y, y') \leq |g(y) - g(y')|_2 \leq \alpha \cdot d(y, y')$, where $\alpha$ is a constant satisfying $\alpha \leq O(\log m)$.

Our training algorithm contains two stages:

- Using Bourgain’s theorem to construct a latent space distribution $\mathcal{Z}$ (mixture of Gaussians).
- Training generative network $G$ with additional distance loss.

Expriements

We define distance loss to be

$$L_{\text{dist}}(G) = \mathbb{E}_{z, z' \sim \mathcal{Z}}[(\log(d(G(z)), G(z'))) - \log(z_i - z'_j)^2].$$

Our new objective is $L(G, D) = L_{\text{gan}}(G, D) + \beta \cdot L_{\text{dist}}(G)$. We still try to minimize it over $G$ and maximize it over $D$.

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